Chapter 9 – **Credit markets, wealth constraints, and allocative inefficiency**

Credit constraints induce the borrower to adopt actions in the interest of the lender (cotton planting in South US).

Additional examples:
① home ownership induces better case of the residence and higher levels of participation in local government activities. However renting prevails among asset-poor families, because they are prevented access to mortgage markets.
② land redistribution may not last once farmers do not get access to credit markets in order to improving farming technology.
③ access to education is prevented by lack of credit.

"In Walrasian setting, the poor are constrained to but less than the rich, but they transact on the same term.... On the contrary, when contracts in financial markets are incomplete or unenforceable, individuals lacking wealth are either precluded from entering in a class of contracts that are available to the wealthy or enter these contracts on unfavourable terms. **Thus wealth differences have qualitatively effects, excluding some and empowering others**".

The main reason is that wealth ownership attenuates the incentive problems arising from contractual incompleteness in principal agent relationship.

Empirical evidence of credit constrainedness:
⇨ increase of credit limits generates an immediate increase in debt → families were prevented access to credit
⇨ increase in family wealth affects the probability to startup a business
⇨ lack of wealth affect farming technology in developing countries
⇨ buying on instalments reveals high intertemporal interest rates

BORROWER AND LENDER

The promise to repay a debt is not enforceable: the borrower choice of risky activities is not freely observable – the borrower may not have funds to repay when repayment is due.

Assumptions:
① all actors are risk neutral (i.e. the utility of an expected outcome is equivalent to the expected utility of outcomes: $E[U(x)] = U(E[x])$ - this is possible only in the class of U being a linear function).

② the return of a risky project is given by

$$r = \mu f(1-f) + f \cdot 0$$

where f is the probability of failure. When successful, the project returns μf, where μ is the quality of the project. An increase in f (up to $\frac{1}{2}$) raises both expected return and its risk (as measured by the variance of return $Var(r) = \frac{\mu^2 f^3(1-f)}{2}$). Think of f as the speed of operating a machine, which has a probability of total failure also equal to f. f is the choice variable of the investor.

③ there is a risk-free alternative yielding ρ

Robinson Crusoe
A self-financing owner of the project would maximise the expected return by selecting

$$\max_f r = \mu f - \mu f^2 \rightarrow \frac{dr}{df} = \mu - \mu 2f = 0 \rightarrow f^* = \frac{1}{2} \rightarrow r = \mu \cdot \frac{1}{2} \cdot \left(1 - \frac{1}{2}\right) = \frac{\mu}{4}$$

Therefore a project is worth undertaking if

$$\frac{\mu}{4} > 1 + \rho$$

Fully contractible (f is fully observable by the lender)

A wealthless agent A borrows 1 unit of capital from a lender P at the cost of δ (including both interest rate and repayment of the principal) which is repaid only if the project is successful (with probability $(1-f)$ - limited liability).

There is a conflict of interest between the agent (who is not concerned with the riskiness of the project but cares of the expected return only) and the principal (who wants to assure the maximum probability of repayment).

The expected return for the agent A is

$$y(f,\delta)=\mu f(1-f)-\delta(1-f)=(\mu f-\delta)(1-f)$$

The best alternative for A is 0, so the participation constraint for A is $y(f,\delta)\geq 0$.

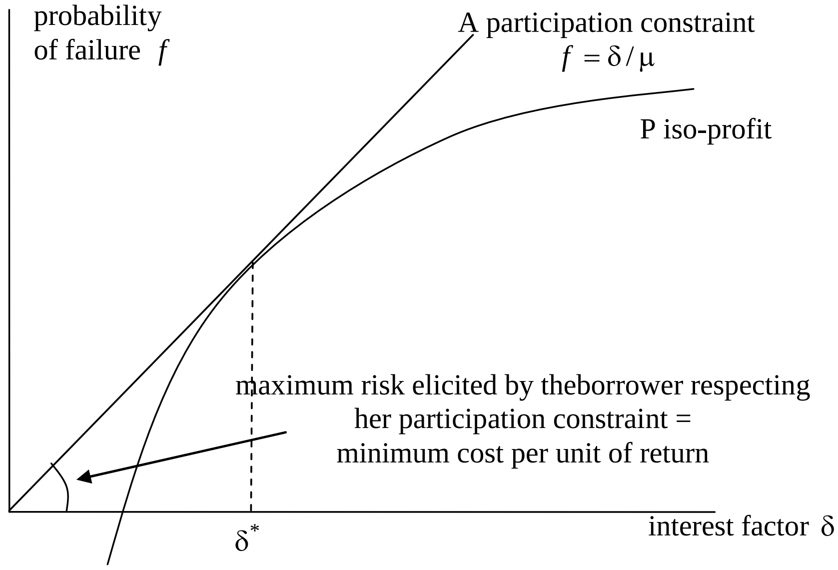
The lender offers a contract yielding the minimum return to the borrower, i.e. sets the repayment equal to

$$\mu f-\delta=0 \quad\rightarrow\quad \delta=\mu f$$

Then her expected profits are given by

$$\pi=\delta(1-f)=\mu f(1-f)$$

Since she can observe the actions undertaken by the borrower, she will impose a selection of f that maximises her expected profit ($f^{*}=\frac{1}{2}$).
The optimal contract (from the view-point of the lender) is: A agrees to implement $f^{*}=\frac{1}{2}$ and to repay an amount $\delta^{*}=\frac{\mu}{2}$ if the project is successful (with probability $(1-f)$). This contract yields $\pi=\frac{\mu}{4}$.
Thus a full contract gives an identical result to the autarky.



Non-contractible risk, no collateral
The lender cannot observe f, which is freely selected by the borrower. According to incentive compatibility, the agent will select f in order to maximise her expected return

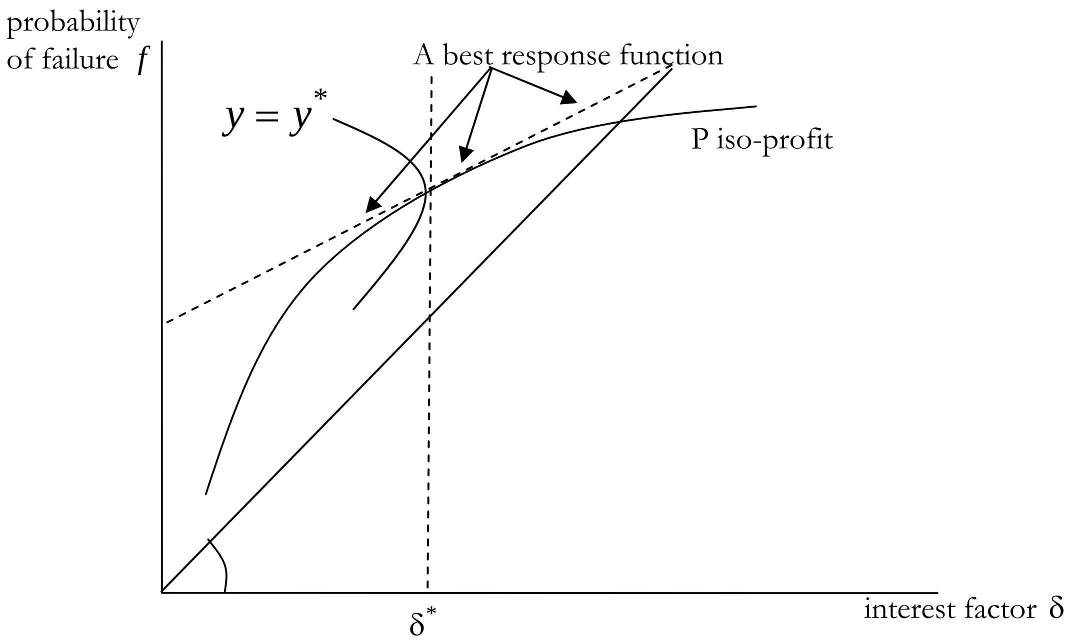
$$\max_f y(f,\delta) \qquad \to \qquad \frac{dy}{df} = \mu(1-2f)+\delta = 0 \to f^* = \frac{\delta+\mu}{2\mu} = \frac{1}{2} + \frac{\delta}{2\mu}$$

The borrower does select a higher level of risk than the autarky case.

The lender now solves the problem

$$\max_\delta \pi = \delta(1-f(\delta)) \qquad \to \qquad \frac{d\pi}{d\delta} = 1 - f - \delta\frac{df}{d\delta} = 1 - \left(\frac{1}{2} + \frac{\delta}{2\mu}\right) - \frac{\delta}{2\mu} = 0$$

yielding $\delta^* = \dfrac{\mu}{2}$, which leads to $f^* = \dfrac{3}{4}$. The agent will implement a higher risk than the full contract case. Since the best response function is above the participation constraint, the borrower enjoys a rent by obtaining a positive return.



Infinite horizon with contingent renewal

The incentive problem is attenuated if the lender offers the borrower a contingent renewal contract over an infinite horizon → she offers a loan (for a single period) with a promise to renew the loan if the project does not fail, and not otherwise.

The agent's expected value is

$$v = \frac{y(f,\delta) + (1-f)v + fz}{1+i} = \frac{y - iz}{i+f} + z$$

Assume for simplicity $z = 0$ and $i = 0$ $\rightarrow$ $v = \dfrac{y}{f} = \dfrac{\mu f(1-f) - \delta(1-f)}{f}$.

The incentive compatibility requires the agent maximising her expected return

$$\max_f v \qquad \rightarrow \qquad \frac{dv}{df} = \frac{-\mu f^2 + \delta}{f^2} = 0 \qquad \rightarrow \qquad f^* = \sqrt{\left(\frac{\delta}{\mu}\right)}$$

If $\delta < \mu$ it is easy to prove that in the infinite horizon yields a lower level of risk, because

$$\sqrt{\frac{\delta}{\mu}} < \frac{1}{2} + \frac{\delta}{2\mu}$$

The principal's expected profit is unaffected. Therefore she will select $\delta^* = \frac{4\mu}{9}$, which ensures $f^* = \frac{2}{3}$.

Thus the use of contingent renewal allows a Pareto improvement over the single case. Here is a summary table of the results.

case	agent's best response function $f^*(\delta,\mu)$	selected risk for the project f^*	interest charged δ^*	expected payoffs (y,π) per period
autarky (R.Crusoe)	-	$\frac{1}{2}$	-	$\frac{\mu}{4}$
contractible risk	$f = \frac{\delta}{\mu}$	$\frac{1}{2}$	$\frac{\mu}{4}$	$0, \frac{\mu}{4}$
non-contractible risk – single period	$f = \frac{1}{2} + \frac{\delta}{2\mu}$	$\frac{3}{4}$	$\frac{\mu}{2}$	$\frac{\mu}{16}, \frac{\mu}{8}$
non-contractible risk – multi-period	$f = \sqrt{\frac{\delta}{\mu}}$	$\frac{2}{3}$	$\frac{4\mu}{9}$	$\frac{2\mu}{27}, \frac{4\mu}{27}$

WEALTH CONSTRAINTS AND THE USE OF COLLATERAL

Human capital cannot be used as collateral because effort is not contractible. Financial capital can be offered as collateral for borrowing.

In addition, if μ is not observable to the lender, the use of the borrower's own wealth can be used as signal of the quality of the project.

The financial involvement of the borrower attenuates the misalignment of objectives between borrower and lender, thus reducing the selected level of risk. It also creates a barrier beyond which borrower without wealth would not be financed.

Non-contractible risk with borrower's equity
If the borrower has $\kappa < 1$ wealth invested in riskless asset yielding ρ as return. When undertaking a risk project she may be requested to invest this wealth in the project in order to obtain $(1-\kappa)$ external financing. Thus the expected return of the project (including foregone income from the riskless asset) is

$$y(f,\delta) = \mu f(1-f) - \delta(1-\kappa)(1-f) - (1+\rho)\kappa$$

The incentive compatibility constraint leads to the selection of

$$\max_f y(f,\delta) \qquad \rightarrow \qquad \frac{dy}{df} = \mu(1-2f) + \delta(1-\kappa) = 0 \rightarrow f^* = \frac{1}{2} + \frac{\delta(1-\kappa)}{2\mu}$$

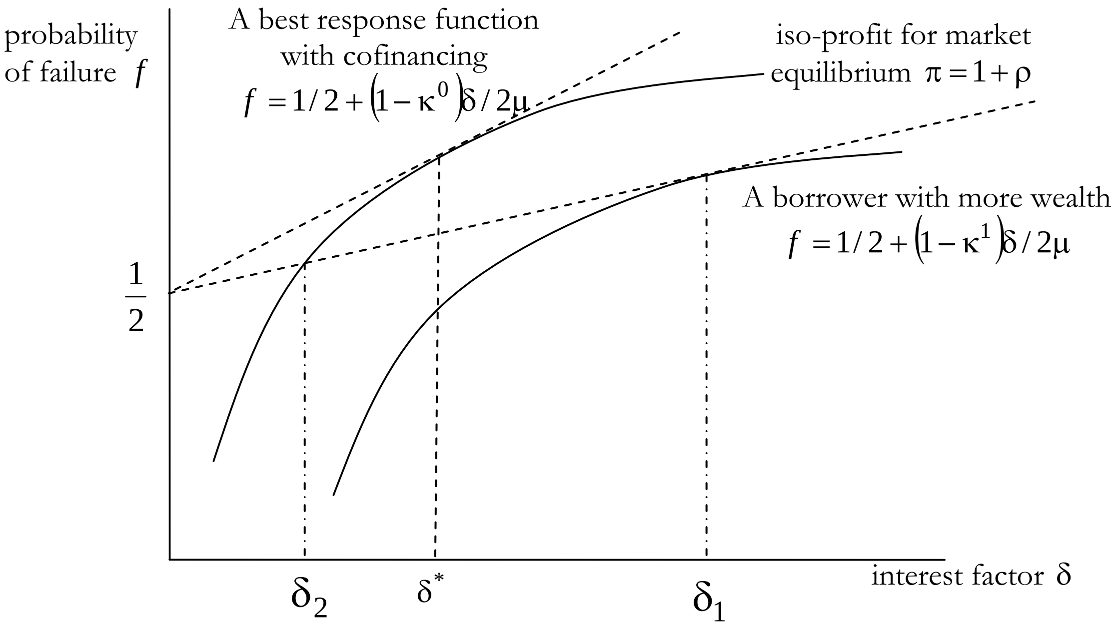
As the equity share of the borrower increases, the chosen level of risk falls. When $\kappa \rightarrow 1$, $f^* \rightarrow \frac{1}{2}$: complete self-financing replicates the prudent social optimum.

The lender, observing the choice of f^*, will set $\delta^* = \frac{\mu}{2(1-\kappa)}$.
Cofinancing reduces the cost of funds.

When going to the general equilibrium, let us impose a zero profit condition for all the lenders

$$\pi = \delta(1 - f) = (1 + \rho)$$

This requires a given level of cofinancing κ^0. For lower level of risk and/or higher cost of funds, profits are positive and new lenders enter the market. For higher level of risk and/or lower cost of funds, profits are negative and lenders do not sing contracts.



Borrowers with a level of wealth below κ^0 will not find any financing (their best response function is above the iso-profit condition) ⇨ market exclusion.

Borrower with a level of wealth above $\kappa^1 > \kappa^0$ create a surplus opportunity, which can be shared between borrower and lender, depending on the institutional set-up:
⇨ if the lender is first mover, she will offer δ_1
⇨ if the borrower is first mover, she will offer δ_2.
Otherwise they will bargain a cost of financing between the two values.

In both cases, the cost of financing is decreasing in the wealth of the borrower (price discrimination).

If two borrowers with different wealth are offered the same cost of financing, the richer one will obtain more funds (quantity discrimination).

Suppose now that quality of project μ varies among borrowers. Let us consider a case where one borrower is almost unable to provide equity ($\kappa_1 \geq 0$), while the other is wealthier ($\kappa_2 > \kappa_1 \geq 0$). The project of the wealthless agent is of superior quality ($\mu_1 > \mu_2$).

If in equilibrium both get funding, it must be the case that both yield the same return

$$\pi_1 = \delta_2\left(\frac{1}{2} - \frac{\delta(1-\kappa_2)}{2\mu_2}\right) = \delta_1\left(\frac{1}{2} - \frac{\delta(1-\kappa_1)}{2\mu_1}\right) = \pi_1$$

If we allow for the same cost is charged to both borrowers ($\delta_1 = \delta_2$), then it must be that

$$\frac{1-\kappa_2}{\mu_2} = \frac{1-\kappa_1}{\mu_1} \quad \Leftrightarrow \quad \frac{\mu_2}{\mu_1} = \frac{1-\kappa_2}{1-\kappa_1}$$

The minimal quality of a project required to secure funding expressed as a ratio among two prospective borrowers is proportional to the fraction of projects that cannot be self-financed → wealthier borrower obtain financing for lower quality projects.

Thus in a competitive equilibrium
① wealthier borrowers can obtain financing for larger projects
① wealthier borrowers can obtain financing for lower quality projects
① for identical size and quality, wealthier borrower pay lower interest rates.

This is inefficient, because poor borrowers with better quality projects could not be financed.

The social optimum requires that no excluded of either borrower be of higher quality than any included project.

However, consider again two borrowers, with different level of wealth and an identical list of projects to be financed. If both obtain funding it must be the case that

$$\frac{\mu_2}{\mu_1} = \frac{1-\kappa_2}{1-\kappa_1}$$

Thus the wealthier borrower obtains more funds than the other, because he achieves a lower level of quality of funded projects.

In this case, redistributing wealth from the rich to the poor (i.e. $\kappa_2 \to \kappa_1$) would lead to the financing of better quality projects, increasing social optimum.

Why we do not observe such a redistribution without taxation ? Because a promise to repay by the poorer borrower to the rich borrower is non enforceable.

Risk aversion and ownership

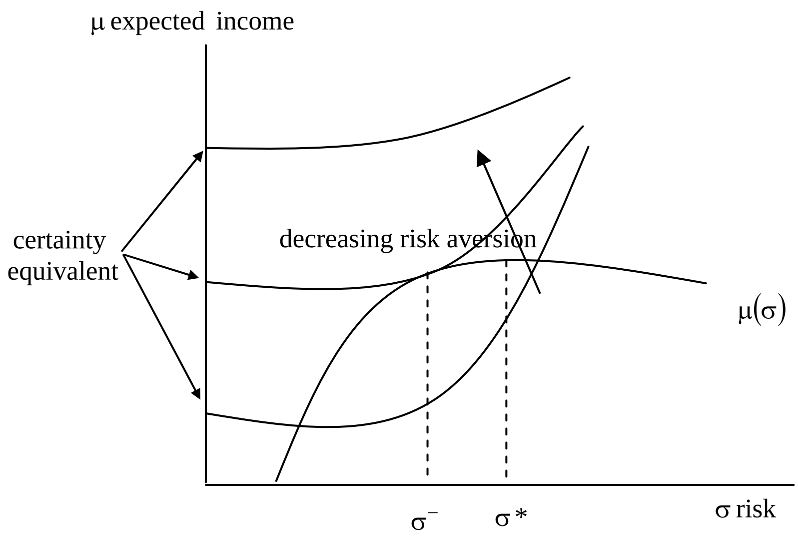
One possible justification for an unequal distribution of wealth is that financially constrained workers/entrepreneurs are more risk adverse than capitalists/bankers $\to$ empirical evidence that risk aversion declines with income/wealth.

Instead of using the standard approach defining the risk aversion using the Arrow-Pratt absolute or relative coefficient of risk aversion $\frac{U''}{U'}$, which describes the curvature of the expected utility function, we define the individual utility function in terms of expected return μ (good) and risk σ (bad).

$$v = v\left(\underset{+}{\mu(\sigma)}, \underset{-}{\sigma}\right)$$

But the expected return is increasing in risk. The return of an investment is given by

$$y = \mu(\sigma) + \sigma\varepsilon, \quad \varepsilon \sim (0,1), \quad Var(y) = \sigma^2$$



The slope of these indifference curves $-\dfrac{v_\sigma}{v_\mu} = \eta$ is the marginal rate of substitution between risk and expected income. The vertical intercept gives the maximum than an individual will pay for the opportunity to draw an income form a distribution with mean and dispersion given by any other point of the indifference locus (*certainty equivalent*).

The risk-return schedule $\mu(\sigma)$ reaches a maximum and then declines for excessive level of risk. A risk neutral agent will optimally select the highest expected income, associated to σ^*. A risk adverse agent will give up some return in exchange of less risk, and will select σ^- by choosing $\mu' = -\dfrac{v_\sigma}{v_\mu}$.

Under what conditions an asset-poor agent prefer to be the owner-operator rather than a wage earner on the same project ?

If wage earner, he needs to be supervised. If κ is the required capital to hire a worker and ρ is the return on the riskless asset (*opportunity cost*), the owner will obtain the following profit

$$\pi(\sigma) = \mu(\sigma) + \sigma\varepsilon - \rho\kappa - w - m$$

where m are supervision costs. If the owner is risk neutral s/he will select σ^*. If there is full entry in the sector, profits will go to zero because wages will go up, and the maximum wage will be

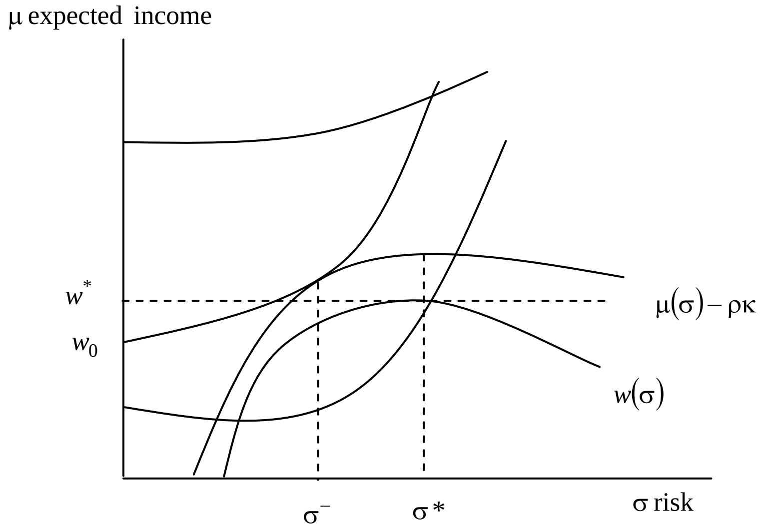
$$\pi(\sigma) = 0 \rightarrow w^* = \mu(\sigma^*) - \rho\kappa - m$$

Would an employee receiving w^* with certainty prefer to be residual claimant on the uncertain income of the project assuming that she can choose the level of risk ? If the capital can be rented for $\rho\kappa$ per period (absence of capital market imperfections), then the owner-operator income will be

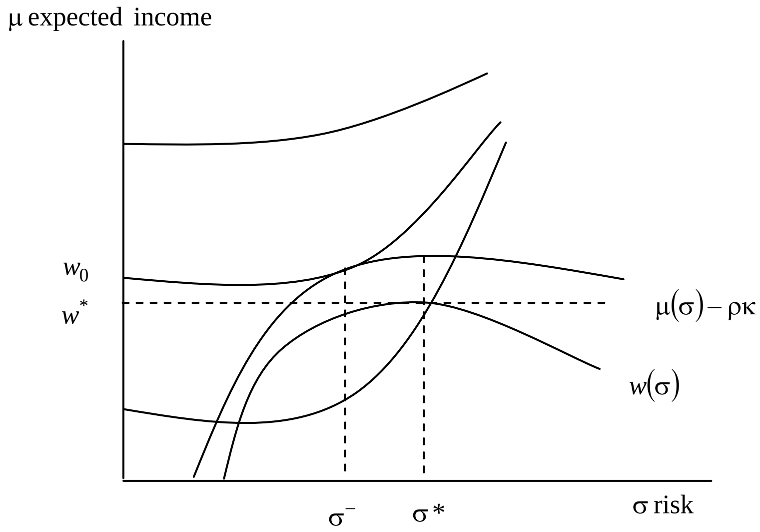
$$y(\sigma) = \mu(\sigma) + \sigma\varepsilon - \rho\kappa \;\rightarrow\; \quad E[y] = \mu(\sigma) - \rho\kappa$$

If the owner-operator is risk-adverse, she will select the level of risk that maximises her utility function $v = v\left(\underset{+}{\mu(\sigma) - \rho\kappa}, \underset{-}{\sigma}\right)$, i.e. satisfying $\mu' = -\frac{v_\sigma}{v_y}$.

Two alternative situations. In both cases the $[\mu(\sigma) - \rho\kappa]$ schedule exceeds the $w(\sigma)$ schedule by the monitoring cost, which is not required since the owner-operator solves the incentive problem by self-monitoring. In both cases risk aversion induce a risk selection σ^- which is inferior to the optimal one σ^*. In the first case the certainty equivalent w_0 of assuming a risk is lower than w^*, thus the employee will prefer to remain employee.



In the second case the employee prefers to become residual claimant despite the risk associated to being owner-operator → we should observe owner-operated projects instead of wage employment → redistribution of property rights is Pareto-improving.



We now relax the assumption of perfect financial markets, replacing with an increasing interest rate for increasing loans

$$r = r\left(\frac{k}{\underline{\kappa}}\right), \quad r(1) = \rho$$

where k is the wealth of the owner-operator. Her expected income becomes

$$E[y] = \mu(\sigma) - r\left(\frac{k}{\kappa}\right)\kappa$$

which is lower than the previous schedule. This makes it more likely the preference for dependent employment. Thus the potential owner-operator will remain employee for both risk aversion and insufficient wealth in presence of imperfect financial markets.

The source of the efficiency gain is the elimination of monitoring costs allowed by the substitution of self employment for wage employment. This gain is partially offset by the reassignment of control of the risk choice from the risk neutral erstwhile owner to a risk-averse owner-operator, couple with the elimination of the efficient insurance against risk provided by the assignment of full residual claimancy to the risk neutral owner.

Redistribution of assets may be more efficient, though they are not Pareto improving → wealth redistribution addresses the incentive problem at the expenses of reduced risk-taking.